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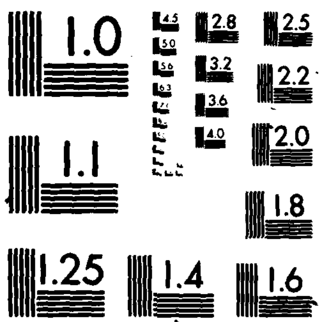
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TECHNICAL REPORT ARLCB-TR-81002

COMPUTATION SCHEMES FOR SENSITIVITY COEFFICIENT OF EXTERIOR BALLISTICS WITH VELOCITY SQUARE DAMPING

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January 1981

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The principal equation of exterior ballistics with velocity square damping term has been integrated analytically in obtaining the solution for tangential velocity in terms of the elevation angle and other parameters. Using the variational method, four equations are obtained. The first one is derived from consideration of terrain slope and the second one is determined by hitting the target. The third and fourth equations are variations of the range and

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20. Abstract (Cont'd)

elevation drag functions, respectively.

The computation involves integrals which can be evaluated analytically if the drag coefficient is relatively small. In simplifying the computational procedure we can assign the launch and impact slopes and then compute the drag functions and the terrain slope. However, this procedure is in reverse order because physically the terrain slope is known a priori the rounds are fired. If the terrain slopes and launch slopes are given first, an iteration procedure in computation is required to solve for the impact slopes. The sensitivity coefficients and the range ratios are then computed and plotted for various terrain slopes and launch slopes as the drag coefficients are varied.

TABLE OF CONTENTS

TABLES

LIST OF ILLUSTRATIONS

INTRODUCTION

The research on sensitivity coefficients on ballistics has been discussed in two previous works. The first paper[1] matches the sensitivity coefficient of exterior ballistics to that of interior ballistics, while the second paper[2] presents the sensitivity coefficient of exterior ballistics with velocity square damping. This report continues the work of the latter[2] and extends the analytic study in order to give numerical solutions of the problem.

The design of a gun involves numerous parameters. These parameters should be in such a combination that the best first round accuracy is given. While the shell leaves the gun it has perturbations for the muzzle elevation angle and the muzzle velocity. The ratio of the two is the sensitivity coefficient of the interior and the exterior ballistics. It is desired to compensate for errors due to uncertain changes of muzzle velocity by the automatic response of the muzzle elevation angle within the gun system. With a correct design this compensation can be made by matching the exterior ballistics to the interior ballistics through the analysis of gun dynamics. This process is called passive control since there is no external measurement involved nor instrumentation needed for control. This general problem can be formulated by first investigating the sensitivity coefficients for exterior ballistics with velocity square damping.

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[1] Shen, C. N., "On the Sensitivity Coefficient of Exterior Ballistics and Its Potential Matching to Interior Ballistics Sensitivity," Proceedings of the Second US Army Symposium on Gun Dynamics at the Institute on Man and Science, Rensselaerville, NY, September 1978, sponsored by USA ARRADCOM.

[2] Shen, C. N., "Sensitivity Coefficient of Exterior Ballistics With Velocity Square Damping," Transactions of the Twenty-Fifth Conference of Army Mathematicians, ARO Report 80-1, January 1980, pp. 267-282.

PREVIOUS WORK

In the second paper the principal equation of exterior ballistics has a drag term which, in this case, is proportional to the square of the velocity in the tangential direction of the projectile. The sensitivity coefficient is expressed as the ratio of the initial elevation angle deviation to the initial precentage velocity deviation. The work is to find analytically the sensitivity coefficient of the exterior ballistics with velocity square damping which comes from the nonlinear air resistance for a projectile. This principal equation is integrated analytically in obtaining the solution for tangential velocity in terms of the elevation angle, together with all the necessary initial conditions. The horizontal range and the vertical range are also expressed as integrals of certain function of the elevation angles. In order to obtain the sensitivity coefficient it is necessary to find the perturbations of the horizontal and vertical ranges. This procedure is similar to that of evaluating differentiation under the integral sign. The perturbation of the ranges is the sum of the perturbations due to the initial velocity, the initial elevation angle and the impact elevation angle. By setting to zeroes the range perturbations we can group the coefficients of the perturbations into two separate equations. The ratio of the perturbations for initial elevation angle to that for initial velocity is the sensitivity coefficient for exterior ballistics that we are seeking.

RESULT OF PAST RESEARCH

The following results are concluded as:

1. The principal equation of exterior ballistics is derived with the trajectory slope as the independent variable.

2. The closed form solution for the horizontal component of trajectory velocity is determined for the case of exterior ballistics with velocity square damping.

3. The nondimensional range is obtained in terms of an end slope function and a range drag function.

4. Variations of the nondimensional range are expressed as variations of launch velocity.

5. Variations of the range drag function are in terms of the variations of the range drag integral.

6. The range drag integral has parameters in the integrand as well as the upper and lower limits. The variations of the integral are found.

7. The partial derivatives of the range drag integrand are evaluated.

8. The variational equation for the range is in terms of elements involving three integrals as coefficients of three variational parameters.

9. The variational parameters are that of launch velocity, the launch elevation angle, and the impact elevation angle.

10. The average of the end slopes is equal to the terrain slope times the range drag function minus the elevation drag function.

11. Variations of the nondimensional elevation are expressed as variations of the end slopes and the variations of the drag function.

12. The variational equations for the elevation are determined similar to that for the range.

13. Eliminating the variations of impact slope, δq_i, from the set of two variational equations gives the ratio of the coefficients of $\delta v_o/v_o$ and $\delta q_o/(q_o-q_i)$.

14. The sensitivity $\delta\theta_o/(\delta v_o/v_o)$ may be obtained by dividing this ratio $\delta q_o/(\delta v_o/v_o)$ by the quantity $(1+q_o^2)$.

However, numerical calculation of this problem was not carried in that paper.[2]

DYNAMICAL EQUATIONS FOR TRAJECTORIES

After several transformations as given in the paper[2] and summarized in Appendix A, the dynamical equations are simplified as

$$du = \frac{c}{q} u^3 (1+q^2)^{1/2} dq \tag{1}$$

$$dx = -\frac{u^2}{g} dq \tag{2}$$

$$dy = -\frac{u^2}{g} q dq \tag{3}$$

In the above equation the independent variable q is the projectile slope which is related to θ as

$$q = \tan\theta \tag{4}$$

and the dependent variable u is related to v and θ as

$$u = v\cos\theta \tag{5}$$

[2]Shen, C. N., "Sensitivity Coefficient of Exterior Ballistics With Velocity Square Damping," Transactions of the Twenty-Fifth Conference of Army Mathematicians, ARO Report 80-1, January 1980, pp. 267-282.

where g = the acceleration due to gravity

c = the drag coefficient of the projectile

v = the velocity of the projectile

θ = the path inclination (elevation angle)

x = the horizontal distance of the projectile

y = the altitude or vertical distance of the projectile

The solution for horizontal component of velocity u can be obtained by integrating Eq. (1) to give

$$\frac{1}{u^2} = \frac{1}{u_o^2}\left\{1-u_o^2\,\frac{c}{g}\,[p(q) - p_o(q_o)]\right\} \tag{6}$$

where

$$p(q) = q(1+q^2)^{1/2} + \ell n[q + (1+q^2)^{1/2}] \tag{7}$$

$$p_o(q_o) = q_o(1+q_o^2)^{1/2} + \ell n[q_o + (1+q_o^2)^{1/2}] \tag{8}$$

$$u_o^2 = v_o^2 \sec^{-2}\theta_o = v_o^2(1+q_o^2)^{-1} \tag{9}$$

and

$$q_o = \tan\theta_o \tag{9a}$$

Finally, Eq. (6) takes the form

$$u^2 = \frac{v_o^2}{1+q_o^2}\left\{1 + \frac{H(q,q_o,v_o^2,c/g)}{1 - H(q,q_o,v_o^2,c/g}\right\} \tag{10}$$

where

$$H(q,q_o,v_o^2,c/g) = \frac{c}{g}\,\frac{v_o^2}{1+q_o^2}\,[p(q) - p_o(q_o)] \tag{11}$$

SOLUTION FOR NONDIMENSIONAL RANGE

In determining the range x for the trajectory the closed form solution of u^2 in Eq. (10) can be substituted into Eq. (2) to obtain the solution in integral form as

$$x_i - x_o = - \frac{v_o^2}{g(1+q_o^2)} [(q_i-q_o) + \int_{q_o}^{q_i} \frac{H(q,q_o,v_o^2,c/g)}{1 - H} dq \qquad (12)$$

where x_i = range at impact point

x_o = range at initial point

and q_i = projectile slope at impact point.

To non-dimensionalize the range, Eq. (12) is divided by the factor v_o^2/g as

$$X(x_i,x_o,v_o)/\Lambda(q_i,q_o) = G_X(q_i,q_o,v_o^2,c/g) \qquad (13)$$

where the nondimensional range is

$$X(x_i,x_o,v_o) = (x_i-x_o)g/v_o^2 \qquad (14)$$

the slope function is

$$\Lambda(q_o,q_i) = \frac{q_o-q_i}{1+q_o^2} \qquad (15)$$

and the range drag function due to air resistance is

$$G_X(q_i,q_o,v_o^2,c/g) = 1 - \frac{1}{q_o-q_i} \int_{q_o}^{q_i} \frac{H(q,q_o,v_o^2,c/g)}{1 - H} dq \qquad (16)$$

VARIATION OF THE NONDIMENSIONAL RANGE AND THE SLOPE FUNCTION

In order to obtain a first round hit of the target one of the conditions is that the variation of the range should be zero, i.e., from Eq. (14)

$$\delta(x_i-x_o) = 0 \qquad (17)$$

We take the perturbation for the nondimensional range from Eq. (14) as

$$\frac{\delta X}{X} = \frac{\delta(x_i-x_o)}{x_i-x_o} - \frac{2\delta v_o}{v_o}$$

$$\frac{\delta X}{X} = 0 - \frac{2\delta v_o}{v_o} \tag{18}$$

The variation of the slope function Λ in Eq. (15) becomes

$$\frac{\delta \Lambda}{\Lambda} = - \frac{\delta q_i}{q_o - q_i} + \frac{\delta q_o}{q_o - q_i} - \frac{2 q_o \delta q_o}{1 + q_o^2} \tag{19}$$

Next, taking the variation of Eq. (13) and using the expressions given in Eq. (18) and (19) we have

$$\frac{\delta X}{X} - \frac{\delta \Lambda}{\Lambda} = \frac{\delta G_x}{G_x} \tag{20}$$

or

$$\frac{\delta G_x}{G_x} = - \frac{2\delta v_o}{v_o} + \frac{\delta q_i}{q_o - q_i} - \frac{\delta q_o}{q_o - q_i} + \frac{2 q_o \delta q_o}{1 + q_o^2} \tag{21}$$

This gives the variation of the range δG_x in terms of the variations of the initial velocity δv_o, and the variations of the initial slope δq_o and that of the impact slope δq_i.

THE SOLUTION FOR ELEVATION

The differential equations for elevation was given in Eq. (3) and the solution for u is in Eq. (10).

Substituting Eq. (10) into Eq. (3) gives

$$dy = - \frac{v_o^2}{g(1+q_o^2)} \left[1 + \frac{H(q, q_o, v_o^2, c/g)}{1 - H}\right] q\,dq \tag{22}$$

Integrating the above one obtains

$$y_i - y_o = - \frac{v_o^2}{g(1+q_o^2)} \left[\frac{q_i^2 - q_o^2}{2} + \int_{q_o}^{q_i} \frac{qH}{1-H}\,dq\right] \tag{23}$$

Rearranging yields the relationship between the range Y, the end slope function Λ, and the elevation drag function G_y.

$$Y(y_i,y_o,v_o)/\Lambda(q_i,q_o) = \frac{1}{2}(q_o+q_i) + G_y(q_i,q_o,v_o^2,c/g) \tag{24}$$

where the nondimensional elevation is

$$Y(y_i,y_o,v_o) = \frac{g(y_i-y_o)}{v_o^2} \tag{25}$$

Λ is given in Eq. (15), and the elevation drag function is

$$G_y(q_i,q_o,v_o^2,c/g) = -\frac{1}{q_o-q_i}\int_{q_o}^{q_i} \frac{qH(q,q_o,v_o^2,c/g)dq}{1-H} \tag{26}$$

TERRAIN SLOPE FROM LAUNCH POINT TO TARGET POINT

If Eq. (25) is divided by Eq. (14) with the aid of Eqs. (16) and (26), one obtains

$$\frac{y_i-y_o}{x_i-x_o} \overset{\Delta}{=} m = \frac{(1/2)(q_o+q_i) + G_y}{G_x} \tag{27}$$

where m is the terrain slope from launch point to target point, a constant parameter. Therefore,

$$(1/2)(q_o+q_i) + G_y = mG_x \tag{28}$$

We use Eq. (27) to find the variational equation for the elevation. Taking the variation of Eq. (28) for any given m, we have

$$(1/2)(\delta q_o+\delta q_i) + \delta G_y - m\,\delta G_x = 0 \tag{29}$$

It is noted that δG_y and δG_x are related in Eq. (29), with also the variations δq_o and δq_i.

VARIATION OF THE RANGE AND ELEVATION DRAG FUNCTIONS

Work was performed on evaluation of the drag function in the previous paper[2] which is summarized in Appendix B. This gives the respective variations of the range and elevation drag functions as

$$\delta G_x = a_v \frac{2\delta v_o}{v_o} + a_i \frac{\delta q_i}{q_o - q_i} + a_o \frac{\delta q_o}{q_o - q_i} \tag{30}$$

$$\delta G_y = b_v \frac{2\delta v_o}{v_o} + b_i \frac{\delta q_i}{q_o - q_i} + b_o \frac{\delta q_o}{q_o - q_i} \tag{31}$$

where

$$a_v = [- \frac{1}{q_o - q_i} I_{12}] \tag{32}$$

$$a_i = [- \frac{H_i}{1 - H_i} - \frac{1}{q_o - q_i} I_{11}] \tag{33}$$

$$a_o = [\frac{2q_o}{1 + q_o^2} I_{12} + \frac{1}{q_o + q_i} I_{11} + (\frac{c}{g}) \frac{v_o^2}{1 + q_o^2} \frac{dp_o}{dq_o} I_{02}] \tag{34}$$

$$b_v = [- \frac{1}{q_o - q_i} J_{12}] \tag{35}$$

$$b_i = [- \frac{q_i H_i}{1 - H_i} - \frac{1}{q_o - q_i} J_{11}] \tag{36}$$

and

$$b_o = [\frac{2q_o}{1 + q_o^2} J_{12} + \frac{1}{q_o - q_i} J_{11} + (\frac{c}{g}) \frac{v_o^2}{1 + q_o^2} \frac{dp_o}{dq_o} J_{02}] \tag{37}$$

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[2]Shen, C. N., "Sensitivity Coefficient of Exterior Ballistics With Velocity Square Damping," Transactions of the Twenty-Fifth Conference of Army Mathematicians, ARO Report 80-1, January 1980, pp. 267-282.

In turn, the integral I_{11}, I_{12}, and I_{02}, and other terms are given as follows.

$$I_{11}(q_o,q_i) = \int_{q_o}^{q_i} \frac{H(q,q_o,v_o^2,c/g)}{1-H} dq \tag{38}$$

$$I_{12}(q_o,q_i) = \int_{q_o}^{q_i} \frac{H}{(1-H)^2} dq \tag{39}$$

$$I_{02}(q_o,q_i) = \int_{q_o}^{q_i} \frac{1}{(1-H)^2} dq \tag{40}$$

$$H_i = H(q=q_i) \tag{41}$$

and

$$\frac{dp_o}{dq_o} = 2(1+q_o^2)^{1/2} \tag{42}$$

Moreover, the integral J_{11}, J_{12}, and J_{02}, and other terms are given as follows.

$$J_{11} = \int_{q_o}^{q_i} qH(q,q_o,v_o^2,c/g)/(1-H)\, dq \tag{43}$$

$$J_{12} = \int_{q_o}^{q_i} qH/(1-H)^2\, dq \tag{44}$$

$$J_{02} = \int_{q_o}^{q_i} q/(1-H)^2\, dq \tag{45}$$

and

$$G_y = -\frac{1}{q_o-q_i} J_{11} \tag{46}$$

SOLUTION FOR SENSITIVITY COEFFICIENT

It is noted that both Eq. (21) and Eq. (30) give the variation δG_x in terms of variations δv_o, δq_i, and δq_o and the equations are independent of each other. Rewriting these equations as:

$$\delta G_x = G_x \left[- \frac{2\delta v_o}{v_o} + \frac{dq_i}{q_o - q_i} - \frac{\delta q_o}{q_o - q_i} + \frac{2q_o\delta q_o}{1+q_o^2}\right] \tag{47}$$

$$\delta G_x = a_v \frac{2\delta v_o}{v_o} + a_i \frac{\delta q_i}{q_o - q_i} + a_o \frac{\delta q_o}{q_o - q_i} \tag{48}$$

Subtracting Eq. (48) from Eq. (47) and then dividing by G_x gives

$$0 = \left[-1 - \frac{a_v}{G_x}\right] \frac{2\delta v_o}{v_o} + \left[1 - \frac{a_i}{G_x}\right] \frac{\delta q_i}{q_o - q_i} + \left[-1 - \frac{a_o}{G_x}\right] \frac{\delta q_o}{q_o - q_i} + \frac{2q_o\delta q_o}{1+q_o^2} \tag{49}$$

It is also noted that Eq. (31) expresses the variation δG_y in terms of variations δv_o, δq_i, and δq_o, and Eq. (29) related δG_y and δG_x with the same set of variations. These are rewritten as

$$\delta G_y = b_v \frac{2\delta v_o}{v_o} + b_i \frac{\delta q_i}{q_o - q_i} + b_o \frac{\delta q_o}{q_o - q_i} \tag{50}$$

$$\delta q_i = - \delta q_o + 2m\delta G_x - 2\delta G_y \tag{51}$$

By substituting Eqs. (48) and (50) into (51) we have

$$\delta q_i = - \delta q_o + 2m\left[a_v \frac{2\delta v_o}{v_o} + a_i \frac{\delta q_i}{q_o - q_i} + a_o \frac{\delta q_o}{q_o - q_i}\right]$$

$$-2\left[b_v \frac{2\delta v_o}{v_o} + b_i \frac{\delta q_i}{q_o - q_i} + b_o \frac{\delta q_o}{q_o - q_i}\right] \tag{52}$$

or

$$0 = \frac{(-2ma_v+2b_v)}{q_o-q_i}\frac{2\delta v_o}{v_o} + \left(1 + \frac{-2ma_i+2b_i}{q_o-q_i}\right)\frac{\delta q_i}{q_o-q_i} + \left(1 + \frac{-2ma_o+2b_o}{q_o-q_i}\right)\frac{\delta q_o}{q_o-q_i} \tag{53}$$

Equations (49) and (53) are in terms of the variations δv_o, δq_i, and δq_o. Sensitivity coefficients are ratio of δq_o and δv_o, thus δq_i can be eliminated by combining these two equations:

$$0 = \left[\left(\frac{-2ma_v+2b_v}{q_o-q_i}\right)\left(1 - \frac{a_i}{G_x}\right) + \left(1 + \frac{a_v}{G_x}\right)\left(1 + \frac{-2ma_i+2b_i}{q_o-q_i}\right)\right]\frac{2\delta v_o}{v_o}$$

$$+ \left[\left(1 + \frac{-2ma_o+2b_o}{q_o-q_i}\right)\left(1 - \frac{a_i}{G_x}\right) + \right.$$

$$\left.\left(1 + \frac{a_o}{Gx} - \frac{2q_o(q_o-q_i)}{1+q_o^2}\right)\left(1 + \frac{-2ma_i+2b_i}{q_o-q_i}\right)\right]\frac{\delta q_o}{q_o-q_i} \tag{54}$$

If we define the ratio S^* as

$$S^* = \frac{\delta q_o/(q_o-q_i)}{\delta v_o/v_o} \tag{55}$$

Then from Eq. (54) this ratio is

$$S^* = (-2)\frac{\left(\frac{-2ma_v+2b_v}{q_o-q_i}\right)\left(1 - \frac{a_i}{G_x}\right) + \left(1 + \frac{a_v}{G_x}\right)\left(1 + \frac{-2ma_i+2b_i}{q_o-q_i}\right)}{\left(1 + \frac{-2ma_o+2b_o}{q_o-q_i}\right)\left(1 - \frac{a_i}{G_x}\right) + \left(1 + \frac{a_o}{G_x} - \frac{2q_o(q_o-q_i)}{1+q_o^2}\right)\left(1 + \frac{-2ma_i+2b_i}{q_o-q_i}\right)} \tag{56}$$

The sensitivity coefficient is defined as

$$S = \frac{\delta q_o/(1+q_o^2)}{\delta v_o/v_o} \tag{57}$$

which give the relationship

$$S = \frac{q_o - q_i}{1 + q_o^2} S^* \tag{58}$$

ITERATION OF SOLUTIONS

The terrain slope m is usually provided before the computation. Equation (27) shows that this terrain slope is in terms of the range drag function, G_x in Eq. (16) and the elevation drag function G_y in Eq. (26). In turn, these drag functions are expressed in terms of the projectile starting slope q_o and its impact slope q_i. Computationally the solution of m for given q_o and q_i is a straight forward substitution procedure using Eqs. (27), (16), and (26). However, since the terrain slope m is known in advance physically and the impact slope q_i is not, we compute instead the impact slope q_i for a given starting slope q_o in Eq. (27). Rewriting this equation we have

$$\begin{aligned} q_i = - q_o &+ \frac{2}{q_o - q_i} \int_{q_o}^{q_i} \frac{qH(q, q_o, v_o^2, c/g)dq}{1 - H} \\ &+ 2m - \frac{2m}{q_o - q_i} \int_{q_o}^{q_i} \frac{H(q, q_o, v_o^2, c/g)dq}{1 - H} \end{aligned} \tag{59}$$

One can guess an initial value for q_i on the right side of the above equation and perform the integration. The value for q_i on the left side can be used as the initial value for q_i on the right side for the second trial. This iteration procedure continues until the value of q_i converges to a numerical solution as its limit.

Equation (59) may be expressed as

$$q_i = - q_o + \frac{2}{q_o - q_i} J_{11} + 2m - \frac{2m}{q_o - q_i} I_{11} \tag{60}$$

where I_{11} and J_{11} are given in Eqs. (38) and (43). These integrals will be evaluated in the next section.

EVALUATION OF INTEGRALS

If the drag coefficient cv_o^2/g is small compared to $(1+q_o^2)$ the denominator terms (1-H) and $(1-H)^2$ in Eqs. (38) through (40) and (43) through (45) may be neglected. One obtains from the above equations the following

$$I_{11} \cong I_{12} \cong \int_{q_o}^{q_i} H dq \tag{61}$$

$$I_{02} \cong q_i - q_o \tag{62}$$

$$J_{11} \cong J_{12} \cong \int_{q_o}^{q_i} qH dq \tag{63}$$

$$J_{02} = \frac{1}{2} (q_i^2 - q_o^2) \tag{64}$$

Equation (11) for H may be rewritten as

$$H = K \frac{1}{1+q_o^2} [p(q) - p_o(q_o)] \tag{65}$$

where

$$K = cv_o^2/g \tag{66}$$

and p(q) is given in Eq. (77). Then the integrals I_{11} and I_{12} are

$$I_{11} = I_{12} = KM \tag{67}$$

where

$$M = \frac{1}{1+q_o^2} [P_{q=q_1} - P_{q=q_o} + (q_o-q_1)p_o(q_o)] \tag{68}$$

and

$$P = \int p(q)dq$$

$$= \frac{1}{3} (1+q^2)^{3/2} + q\ell n[q + (1+q^2)^{1/2}] - (1+q^2)^{1/2} \tag{69}$$

Similarly the integrals J_{11} and J_{12} are

$$J_{11} = J_{12} = KN \tag{70}$$

where

$$N = \frac{1}{1+q_o^2} [Q_{q=q_1} - Q_{q=q_o} + \frac{1}{2} (q_o^2-q_1^2)p_o(q_o)] \tag{71}$$

and

$$Q = \int qp(q)dq$$

$$= \frac{1}{4} q(1+q^2)^{3/2} - \frac{3}{8} q(1+q^2)^{1/2} + (\frac{1}{2} q^2 + \frac{1}{8})\ell n[q + (1+q^2)^{1/2}] \tag{72}$$

In the last section the iteration equation (59) for the solution for q_1 now becomes

$$q_1 = - q_o + 2(mG_x-G_y) \tag{73}$$

where

$$G_x = 1 - (q_o-q_1)^{-1} KM \tag{74}$$

and

$$G_y = - (q_o-q_1)^{-1} KN \tag{75}$$

We are now able to compute q_1 if the quantities m and q_o are given.

The coefficients used in the variation of range and elevation drag functions in Eqs. (32) through (37) give

$$a_v = -(q_o - q_i)^{-1} KM \tag{76}$$

$$a_i = K\{-[p(q_i) - p_o(q_o)] - (q_o - q_i)^{-1} M\} \tag{77}$$

$$a_o = K\{[\frac{2q_o}{1+q_o^2} + (q_o - q_i)^{-1}]M - \frac{dp_o}{dq_o}(q_o - q_i)\} \tag{78}$$

$$b_v = -(q_o - q_i)^{-1} KN \tag{79}$$

$$b_i = K\{-q_i[p(q_i) - p_o(q_o)] - (q_o - q_i)^{-1} N\} \tag{80}$$

and

$$b_o = K\{[\frac{2q_o}{1+q_o^2} + (q_o - q_i)^{-1}]N - \frac{1}{2}(q_o^2 - q_i^2)\frac{dp_o}{dq_o}\} \tag{81}$$

With q_o and q_i known, we are able to compute the coefficients of three a's and three b's as given in the above equations. It is now ready to substitute all the coefficients into Eq. (56) to compute S*. By Eq. (58), in turn, one obtains S, the sensitivity we are seeking.

DETERMINATION OF THE EXTREMAL VALUE OF THE DAMPING COEFFICIENT

The assumption that the denominator terms (1-H) may be dropped in evaluating the integrals depends on

$$H \ll 1 \tag{82}$$

From Eq. (65) this is equivalent to

$$H(q) = K\{\frac{1}{1+q_o^2}[p(q) - p_o(q_o)]\} \ll 1 \tag{83}$$

where

$$p(q) = \{q(1+q^2)^{1/2} + \ln[q + (1+q^2)^{1/2}]\} \tag{84}$$

It is noted that

$$H(q_o) = 0 \text{ at } q = q_o \tag{85}$$

The extremal value of $H(q)$ can be evaluated at $q = q_1$

$$|H| < |H|_{extremal} = |H(q_1)| = K \frac{1}{1+q_o^2} [p(q_1) - p_o(q_o)]| << 1 \tag{86}$$

The above is conservative in that we use the largest value of $|H|$ for determining the range spread of K. For example if $q_o = 1$ and $q_1 = -1$, then

$$\frac{1}{1+q_o^2} p_o(q_o) = (1+\ln 2.414)/2 = 0.940 \tag{87}$$

$$\frac{1}{1+q_o^2} p(q_1) = (-1+\ln 0.414)/2 = -0.940 \tag{88}$$

then

$$K|-0.940 - 0.940| << 1 \tag{89}$$

or

$$K << 0.532 \tag{90}$$

and if $q_o = \sqrt{3}$, and $q_1 = -\sqrt{3}$, then

$$\frac{1}{1+q_o^2} p_o(q_o) = [(1.732)(2) + \ln 3.732]/4 = 1.195 \tag{91}$$

$$\frac{1}{1+q_o^2} p(q_1) = [(-1.732)(2) + \ln 0.268]/4 = -1.195 \tag{92}$$

then

$$K|-1.195 - 1.195| << 1 \tag{93}$$

$$K \ll 0.418 \tag{94}$$

The maximum of K we chose in this paper is

$$\max(cv_o^2/g) = \max K = 0.20 \tag{95}$$

which satisfies the above requirements for the approximation used in evaluating the integrals.

COMPUTATION PROCEDURE

The computation procedure consists of two parts, the iteration procedure for the impact slope q_i and the sensitivity and range computation after obtaining q_i.

(1) Iteration Procedure - By giving the launch slope q_o one can compute such quantities as $p_o(q_o)$ from Eq. (7), $P_{q=q_o}$ from Eq. (69) and $Q_{q=q_o}$ from Eq. (72). The iteration procedure starts by assuming an initial impact slope q_i. Then we can obtain $P_{q=q_i}$ and $Q_{q=q_i}$ from Eqs. (69) and (72), respectively. In addition, we can compute M and N from Eqs. (68) and (71), respectively. By assuming the value of K (i.e., cv_o^2/g) in Eq. (66) the integrals I_{11} and J_{11} are calculated from Eqs. (67) and (70), respectively. With these integrals one can find a new value for the impact slope q_i by using Eq. (60). This iteration procedure continues until the value of q_i converges to a numerical solution as its limit.

(2) Sensitivity and Range Computation - With the iterated solution of the impact slope q_i known, one can proceed to find the sensitivity coefficients and the range. We start to find the range drag function G_x and the elevation drag function G_y from Eqs. (74) and (75), respectively. Next the coefficients a_v, a_i, a_o, b_v, b_i, and b_o can be evaluated by Eqs. (76) through (81).

Substituting these functions and coefficients into Eq. (56) we can get S*, thus the sensitivity coefficient S can be obtained readily from Eq. (58).

The range X can also be computed by combining Eqs. (13), (15), and (74).

NUMERICAL SOLUTIONS

We are using the nondimensional damping coefficient $K = cv_o^2/g = 0.2$ for illustration purpose. The terrain slopes are assumed to be $m = 0$ and $m = 0.2679$ which is corresponding to a terrain angle of 0° and 15° respectively. The launch angle θ_o are varied as follows:

$$\text{for } m = 0,\ 7.5° < \theta_o < 82.5° \text{ at intervals of } 7.5°$$

$$\text{for } \tan^{-1} m = 15°,\ 22.5 < \theta_o < 75° \text{ at intervals of } 7.5°$$

The results are shown in Tables I and II, and plotted in Figures 2 through 6. Figure 2 gives the iterated impact angles for various launch angles. The effect of K to the range is fairly large as shown in Figures 3 and 4. However, the effect of the nondimensional drag coefficient K to the sensitivity coefficient is <u>not</u> very large except near the critical sensitivity which magnitude is unbounded as shown in Figures 5 and 6.

CONCLUSION

This report is the third of a sequence. The findings are summarized as follows.

From the first paper[1] we have:

(1) The error increments in hitting a target are derived and set to zero for a bull's eye landing.

(2) The sensitivity coefficient is defined as the ratio of increments of the initial elevation angle to the increments of the natural logarithm of the velocity.

(3) Without air resistance or damping the maximum range is found to be located at an optimal initial elevation angle, which is half of the sum of 90° and the terrain angle.

(4) Without damping the sensitivity coefficients become infinite at maximum range and zero at minimum range of zero.

(5) The sensitivity coefficient is positive for high trajectories (above 45° initial elevation angle) and negative for low trajectories (below 45° initial elevation angle).

(6) Due to uncertainty of muzzle velocity deviation a gun may be designed by matching the interior ballistics to the exterior ballistics. This is to give automatically and instantly a proportional deviation of its initial elevation angle when the shell leaves the muzzle during firing for the first round.

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[1] Shen, C. N., "On the Sensitivity Coefficient of Exterior Ballistics and Its Potential Matching to Interior Ballistics Sensitivity," Proceedings of the Second US Army Symposium on Gun Dynamics at the Institute on Man and Science, Rensselaerville, NY, September 1978, sponsored by USA ARRADCOM.

The second paper[2] investigated the problem when the principal equation of exterior ballistics has a drag term which is proportional to the square of velocity. The paper is to find analytically the sensitivity coefficient. A sequence of nonlinear transformation was used before the sensitivity problem can be studied. The variation of various parameters is investigated, including the variation under an integral sign.

The present report continues the work of the second and converts it into a different analytical form that is suitable for numerical computation. The solution for sensitivity coefficients is condensed into a single formula with the terms readily computable from a set of parameters. An iteration procedure is used in the computation. Some integrals are approximated in order to obtain their analytical solution forms.

The effect of velocity square damping to the range can be large, especially at the maximum range. However, the effect of velocity square damping to the sensitivity coefficient is usually small, except near the maximum range.

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[2] Shen, C. N., "Sensitivity Coefficient of Exterior Ballistics With Velocity Square Damping," Transactions of the Twenty-Fifth Conference of Army Mathematicians, ARO Report 80-1, January 1980, pp. 267-282.

TABLE I. RANGE AND SENSITIVITY COEFFICIENTS FOR FLAT TERRAIN

For m = 0 θ_o	q_o	$\frac{cv_o^2}{g}$	θ_1(degree)	q_1	X	S
7.5°	0.13165	0	- 7.5000	-0.131652	0.258819	-0.267949
		0.20	- 7.7808	-0.136641	0.249773	-0.268020
15.0°	0.26794	0	-15.0000	-0.267949	0.500000	-0.577350
		0.20	-16.1583	-0.289738	0.465538	-0.578836
22.5°	0.41421	0	-22.5000	-0.414213	0.707106	-1.000000
		0.20	-25.0595	-0.467574	0.636169	-1.011472
30.0°	0.57735	0	-30.0000	-0.577350	0.866025	-1.732050
		0.20	-34.2094	-0.679841	0.755507	-1.804402
37.5°	0.76732	0	-37.5000	-0.767326	0.965925	-3.732050
		0.20	-43.1708	-0.938106	0.821550	-4.422821
45.0°	1.00000	0	-45.0000	-1.000000	1.000000	∞
		0.20	-51.4995	-1.257153	0.835103	14.3160
52.5°	1.30322	0	-52.5000	-1.303225	0.965925	3.732050
		0.20	-58.9394	-1.660305	0.798363	2.645294
60.0°	1.73205	0	-60.0000	-1.732105	0.866025	1.732050
		0.20	-65.5374	-2.198108	0.714006	1.342668
67.5°	1.73205	0	-67.5000	-2.414213	0.707106	1.000000
		0.10	-69.0956	-2.618147	0.647751	0.915074
		0.20	-71.6039	-3.006806	0.585294	0.796916
75.0°	3.73205	0	-75.0000	-3.732050	0.500000	0.577350
		0.10	-76.0085	-4.013342	0.459463	0.529802
		0.20	-77.5372	-4.524636	0.417197	0.465626
82.5°	7.59575	0	-82.5000	-7.595754	0.258819	0.267949
		0.10	-82.9614	-8.099269	0.238697	0.245899
		0.20	-83.6382	-8.969194	0.217891	0.217213

TABLE II. RANGE AND SENSITIVITY COEFFICIENTS FOR 15° TERRAIN

For m = +0.2679 θ_o	q_o	$\frac{cv_o^2}{g}$	θ_1(degree)	q_1	X	S
22.5°	0.41421	0	6.932362	0.121586	0.249772	-0.278596
		0.10	6.783131	0.118944	0.245353	-0.278737
		0.20	6.619309	0.116045	0.240893	-0.278891
30.0°	0.57735	0	- 2.379286	-0.041550	0.464175	-0.634112
		0.10	- 3.002055	-0.052443	0.448368	-0.636174
		0.20	- 3.751089	-0.065562	0.432262	-0.638617
37.5°	0.76732	0	-13.03583	-0.231526	0.628688	-1.214643
		0.10	-14.38786	-0.256530	0.598311	-1.230168
		0.20	-16.14129	-0.289416	0.567092	-1.250663
45.0°	1.00000	0	-24.90070	-0.464200	0.732100	-2.732736
		0.10	-26.98236	-0.509137	0.688258	-2.877936
		0.20	-29.80982	-0.572933	0.642910	-3.107193
52.5°	1.30322	0	-37.50354	-0.767425	0.767364	∞
		0.10	-39.96784	-0.838143	0.714885	32.535621
		0.20	-43.37056	-0.944680	0.660410	13.127275
60.0°	1.73205	0	-50.10622	-1.196250	0.732075	2.731708
		0.10	-52.42571	-1.299732	0.678076	2.405832
		0.20	-55.62687	-1.461933	0.622002	2.031230
67.5°	2.41421	0	-61.97076	-1.878413	0.628640	1.214358
		0.10	-63.73921	-2.026837	0.580688	1.105513
		0.20	-66.17359	-2.264476	0.531007	0.968816
75.0°	3.73205	0	-72.62684	-3.196250	0.464108	0.633962
		0.10	-73.71086	-3.422142	0.428564	0.581436
		0.20	-75.20812	-3.787022	0.391874	0.514330
82.5°	7.59575	0	-81.93802	-7.059954	0.249690	0.278489
		0.10	-82.41359	-7.508239	0.230773	0.255652
		0.20	-83.41359	-8.238227	0.211307	0.226622

REFERENCES

1. Shen, C. N., "On the Sensitivity Coefficient of Exterior Ballistics and Its Potential Matching to Interior Ballistics Sensitivity," Proceedings of the Second US Army Symposium on Gun Dynamics at the Institute on Man and Science, Rensselaerville, NY, September 1978, sponsored by USA ARRADCOM.
2. Shen, C. N., "Sensitivity Coefficient of Exterior Ballistics With Velocity Square Damping," Transactions of the Twenty-Fifth Conference of Army Mathematicians, ARO Report 80-1, January 1980, pp. 267-282.

APPENDIX A

DYNAMICAL EQUATIONS FOR TRAJECTORIES AND VARIABLE TRANSFORMATIONS

For a constant mass travelling in a vertical plane with no life and applied thrust, but having drag and velocity vectors contained in the plane of symmetry as shown in Figure 1, the dynamical equations of motion are:[1]

$$\frac{dx}{dy} - v \cos \theta = 0 \tag{A1}$$

$$\frac{dx}{dt} - v \sin \theta = 0 \tag{A2}$$

$$m\left(g \cos \theta + v \frac{d\theta}{dt}\right) = 0 \tag{A3}$$

$$\frac{d^2x}{dt^2} = - \frac{D \cos \theta}{m} \tag{A4}$$

It is noticed that deviations due to anomalies in the azimuth direction are not considered here.

By differentiating Eq. (A1) with respect to t one obtains

$$\frac{d^2x}{dt^2} = \frac{d}{dt}(v \cos \theta) \tag{A5}$$

Substituting into Eq. (A4) we have

$$\frac{d}{dt}(v \cos \theta) = - \frac{D \cos \theta}{m} \tag{A6}$$

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[1] Shen, C. N., "On the Sensitivity Coefficient of Exterior Ballistics and Its Potential Matching to Interior Ballistics Sensitivity," Proceedings of the Second US Army Symposium on Gun Dynamics at the Institute on Man and Science, Rensselaerville, NY, September 1978, sponsored by USA ARRADCOM.

Solving for $d\theta/dt$ in Eq. (A3) one obtains

$$\frac{d\theta}{dt} = \frac{-g \cos \theta}{v} \tag{A7}$$

Equation (A7) indicates that the differential equations can be transformed from the time domain in t to the angle domain in θ. Equations (A6), (A1), and (A2) are divided by Eq. (A7) in achieving this transformation as

$$\frac{d(v \cos \theta)}{d\theta} = \frac{Dv}{mg} \tag{A8}$$

$$\frac{dx}{d\theta} = - \frac{v^2}{g} \tag{A9}$$

$$\frac{dy}{d\theta} = - \frac{v^2}{g} \tan \theta \tag{A10}$$

Equation (A8) is called the prinicpal equation of exterior ballistics.[2] It can be integrated if the drag D is a known function of velocity v.

The head wind drag is a velocity square damping term given as

$$D = mcv^2 \tag{A11}$$

where

$$c = c_w(\frac{\pi}{4} d^2)(\rho/2) \tag{A12}$$

c_w = the dimensionless resistant coefficient

d = the diameter of projectile

and ρ = the air density.

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[2]Shen, C. N., "Sensitivity Coefficient of Exterior Ballistics With Velocity Square Damping," Transactions of the Twenty-Fifth Conference of Army Mathematicians, ARO Report 80-1, January 1980, pp. 267-282.

Thus the principal equation of exterior ballistics (Eq. (A8)) becomes

$$\frac{d}{d\theta}(v \cos \theta) = \frac{cv^3}{g} \tag{A13}$$

A further transformation of the dependent variable in necessary by letting

$$u = v \cos \theta \tag{A14}$$

where u is the horizontal component of the projectile velocity. Then the dynamical Eqs. (A13), (A9), and (A10) become

$$\frac{du}{d\theta} = \frac{c}{g} u^3 \sec^3 \theta \tag{A15}$$

$$\frac{dx}{d\theta} = - \frac{u^2}{g} \sec^2 \theta \tag{A16}$$

$$\frac{dy}{d\theta} = - \frac{u^2}{g} \sec^2 \theta \tan \theta \tag{A17}$$

To simplify further the form of the dynamical equations another transformation of the independent variable is made by letting

$$\frac{dq}{d\theta} = \sec^2 \theta = 1+q^2 \tag{A18}$$

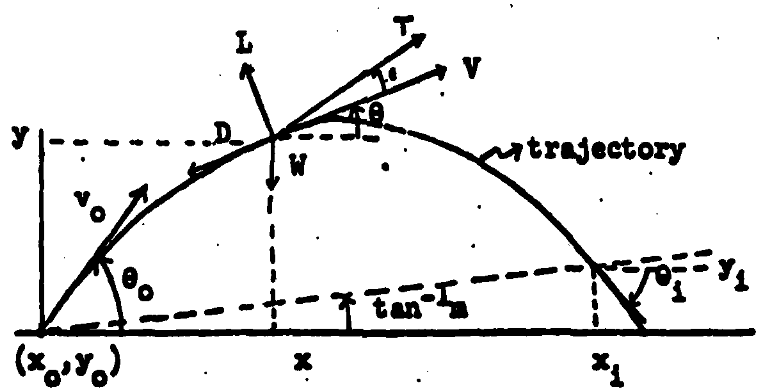

Fig. 1. Forces, Slopes, and Initial and Final Parameters for a Trajectory

APPENDIX B

VARIATION OF THE DRAG FUNCTIONS

As given by Eq. (54) in the previous paper[2] the variation of the range drag function δG_x is

$$\delta G_x = \Big[- \frac{I_{12}}{q_o - q_i} \frac{2\delta v_o}{v_o}$$

$$+ \; - \frac{H_{q=q_i}}{1 - H_{q=q_i}} \frac{I_{11}}{q_o - q_i} \Big] \frac{\delta q_i}{q_o - q_i}$$

$$+ \Big[\frac{2 q_o I_{12}}{1 + q_o^2} + \frac{I_{11}}{q_o - q_i} + \frac{c}{g} \frac{v_o^2}{1 + q_o^2} \frac{dp_o I_{02}}{dq_o} \Big] \frac{\delta q_o}{q_o - q_i} \qquad \text{(B1)}$$

It is noted that the difference of the end slope is not zero, i.e., $q_o - q_i \neq 0$. Therefore, the problem does not become singular. We have expressed the variation δG_x in terms of the variational parameters.

As given by Eq. (77) in the previous paper[2] the variation of the elevation drag function δG_y is

$$\delta G_y = \Big[- \frac{J_{12}}{q_o - q_i} \Big] \frac{2\delta v_o}{v_o}$$

$$+ \Big[- \frac{q_i H_{q=q_i}}{1 - H_{q=q_i}} - \frac{J_{11}}{q_o - q_i} \Big] \frac{\delta q_i}{q_o - q_i}$$

$$+ \Big[\frac{2 q_o J_{12}}{1 + q_o^2} \; \frac{J_{11}}{q_o - q_i} \; \frac{c}{g} \; \frac{v_o^2}{1 + q_o^2} \; \frac{dp_o J_{02}}{dp_o} \Big] \frac{\delta qo}{q_o - q_i}$$

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[2] Shen, C. N., "Sensitivity Coefficient of Exterior Ballistics With Velocity Square Damping," Transactions of the Twenty-Fifth Conference of Army Mathematicians, ARO Report 80-1, January 1980, pp. 267-282.

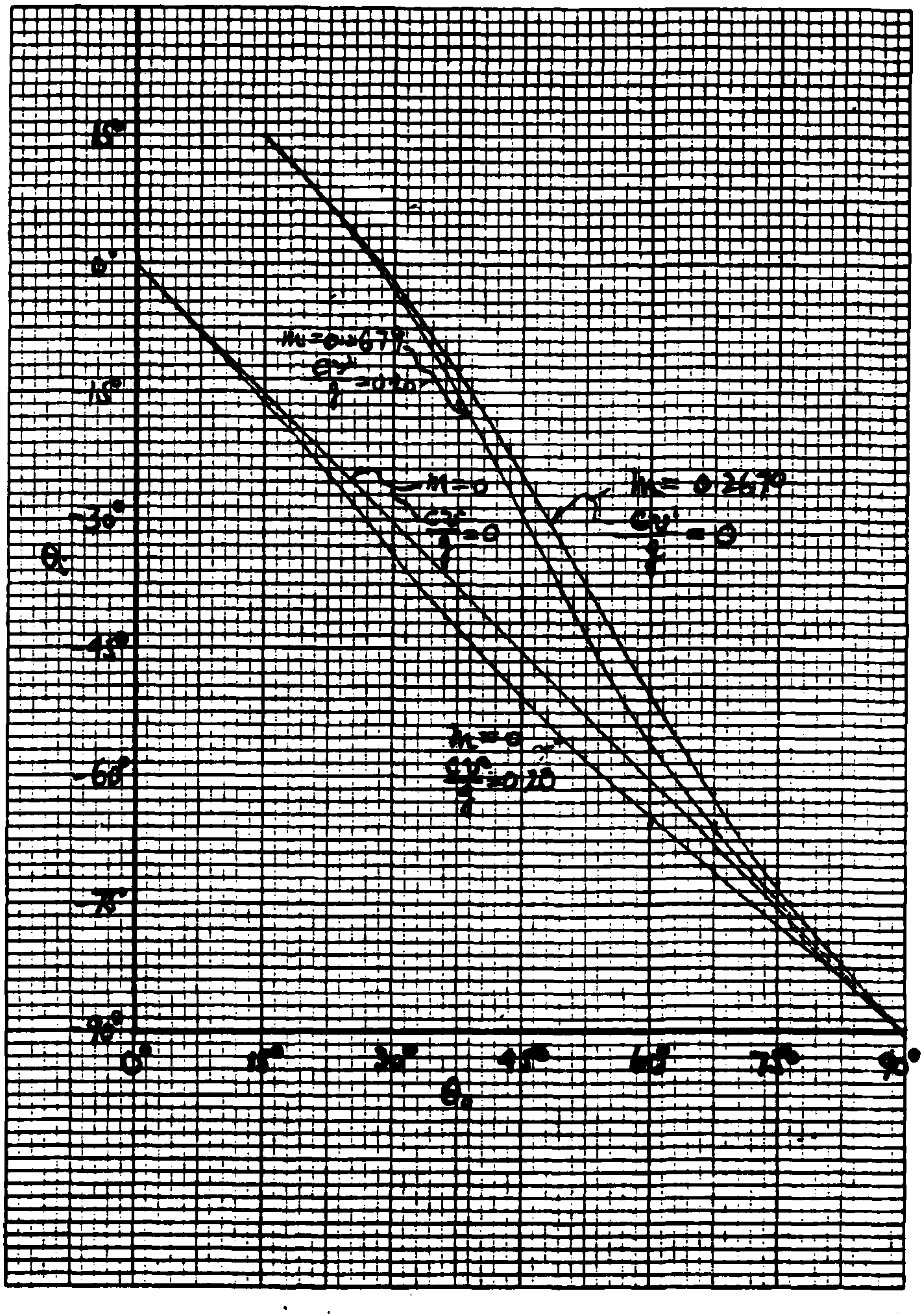

Figure 2. Impact Angle vs. Launch Angle for Different Target Slopes and Damping Coefficients.

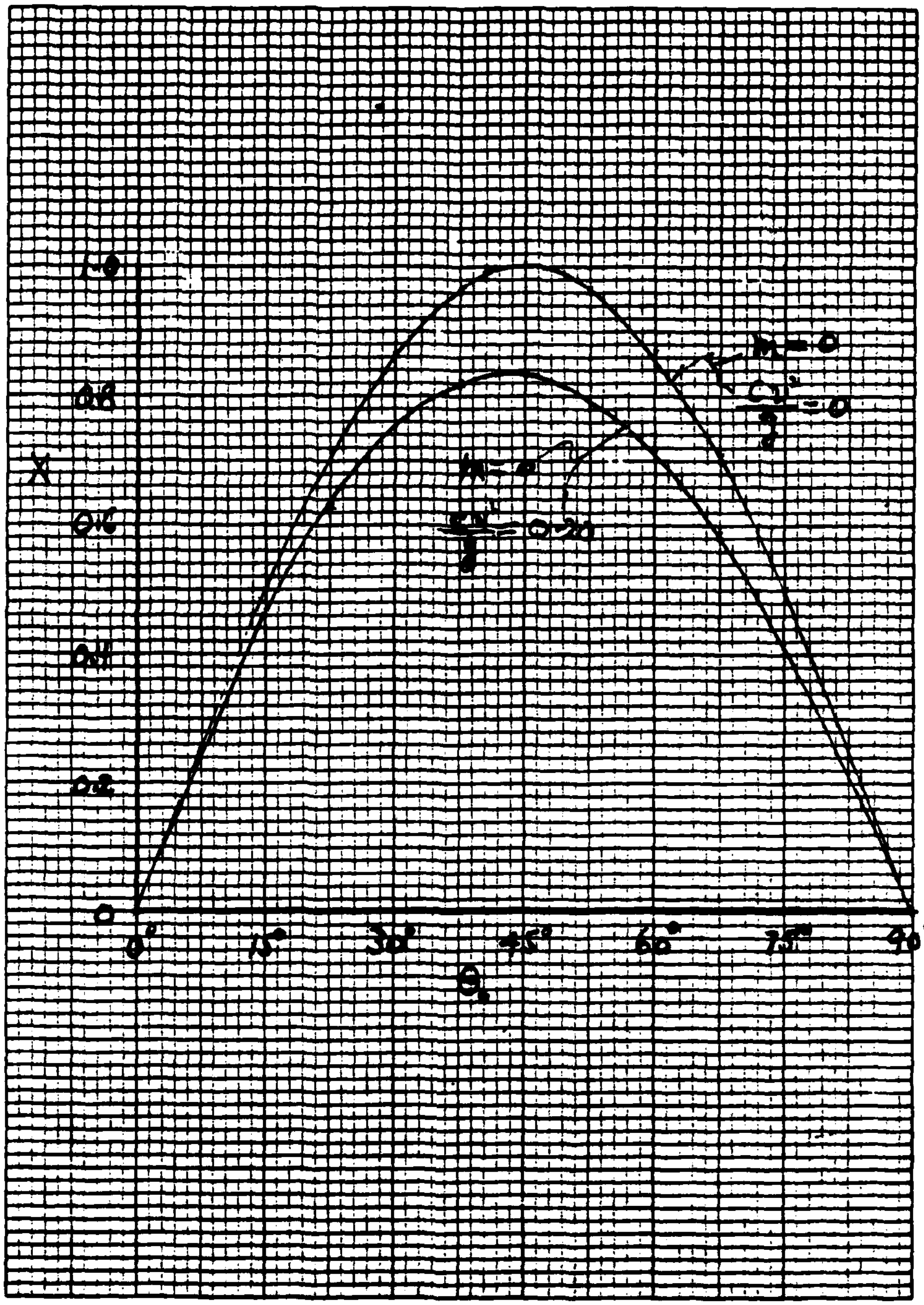

Figure 3. Nondimensional Range vs. Launch Angle for Horizontal Target and Different Damping Coefficients.

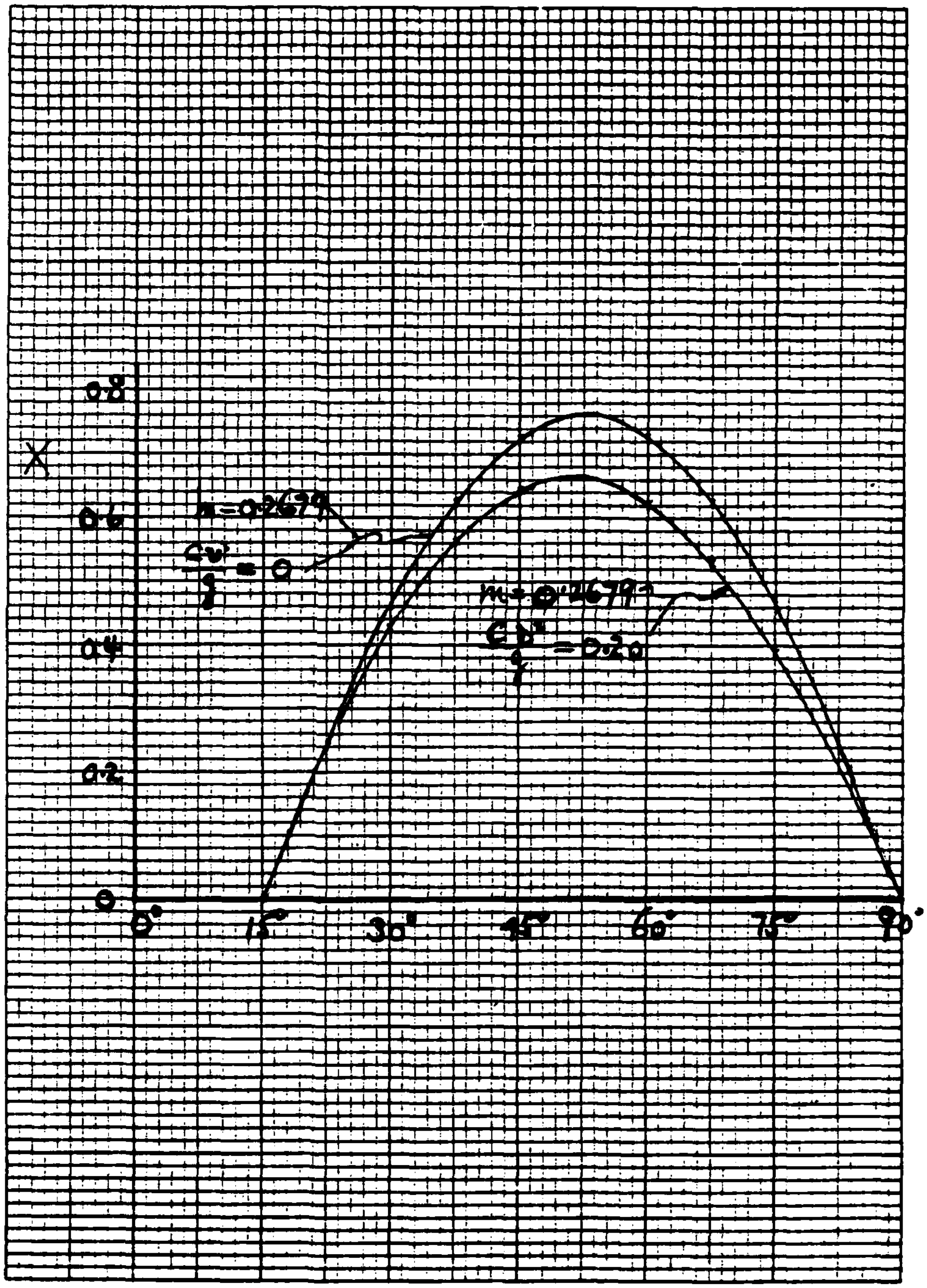

Figure 4. Nondimensional Range vs. Launch Angle for Upgrade Targets and Different Damping Coefficients.

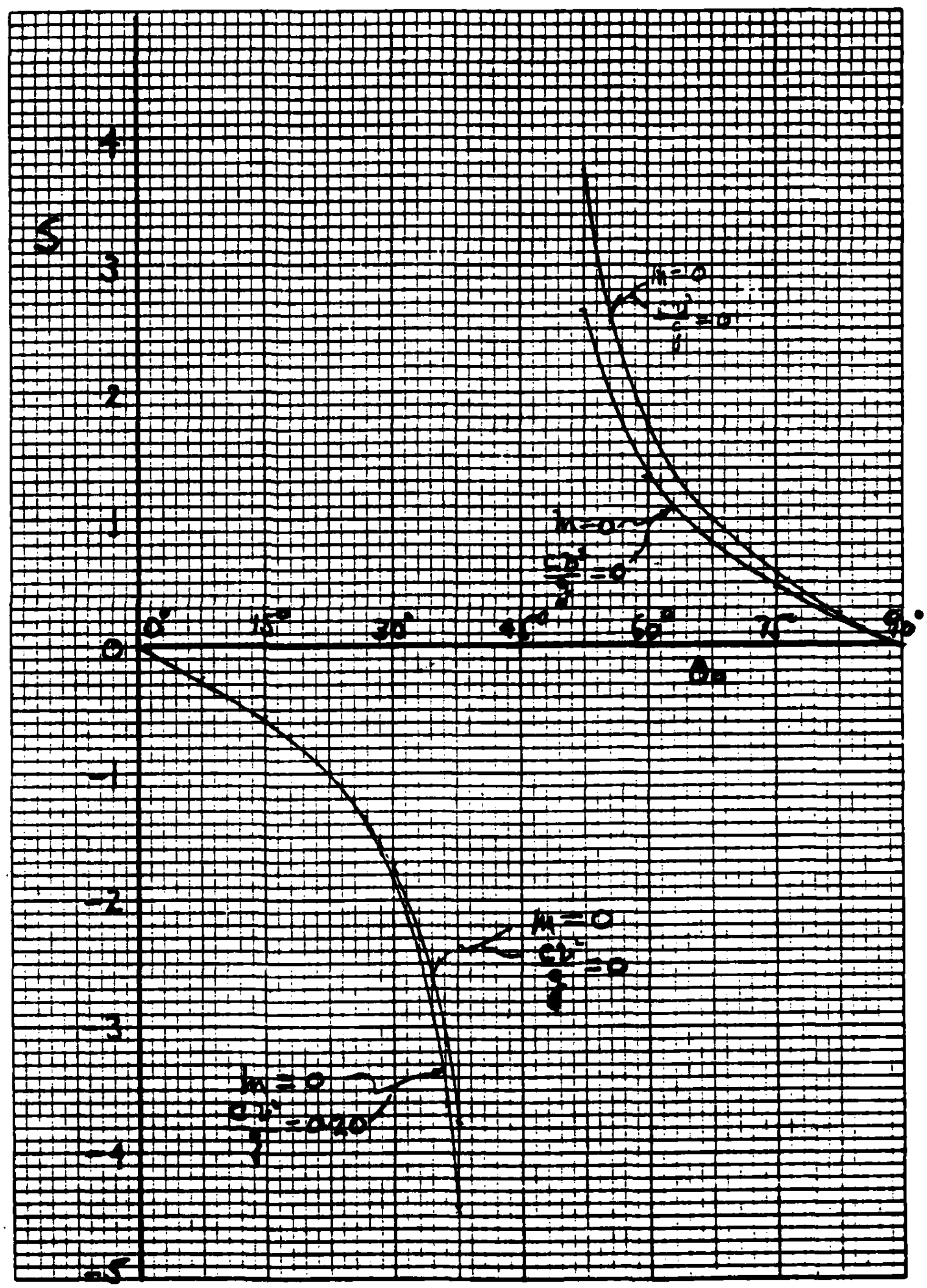

Figure 5. Sensitivity Coefficient vs. Launch Angle for Horizontal Target and Different Damping Coefficients.

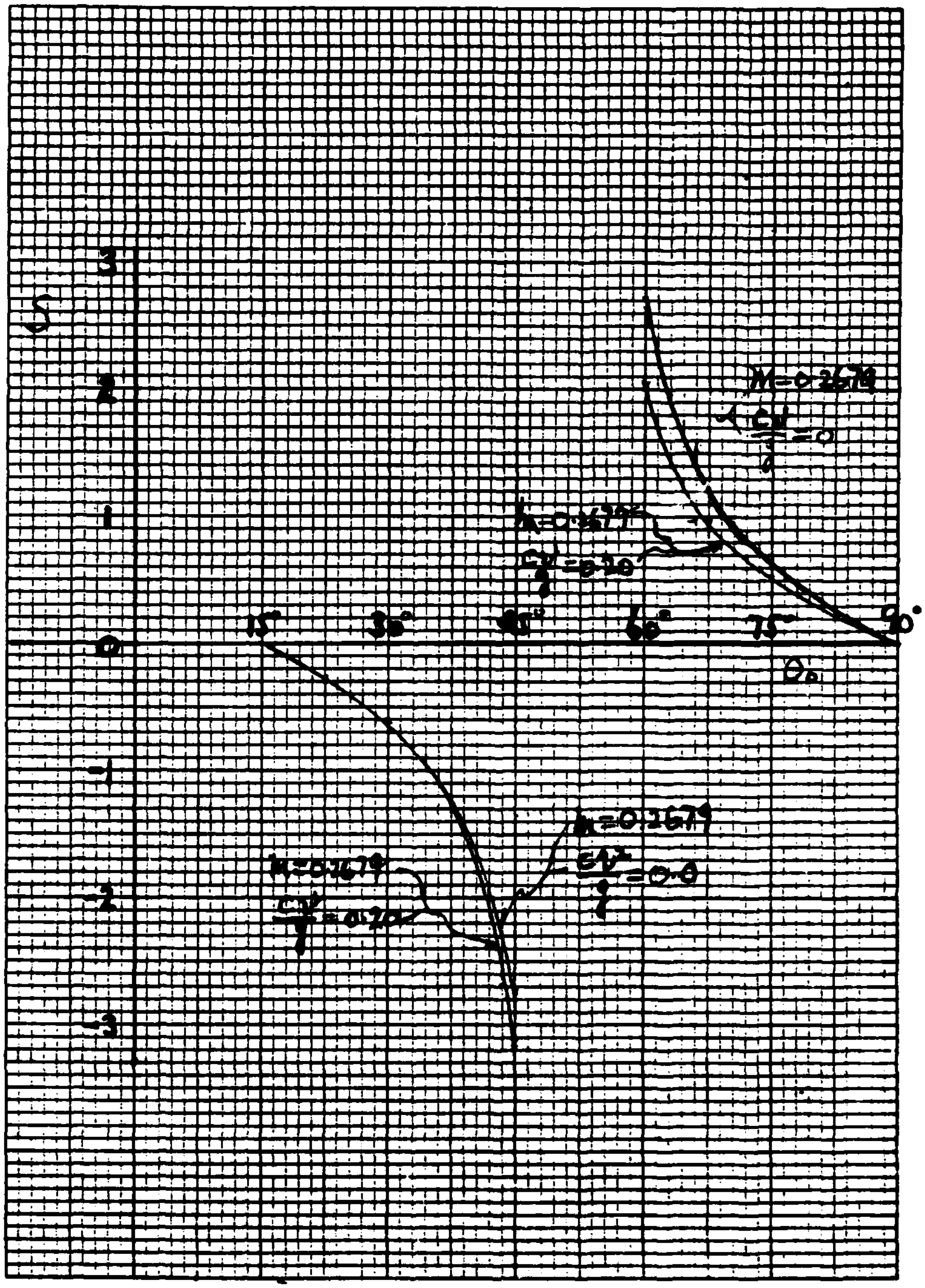

Figure 6. Sensitivity Coefficient vs. Launch Angle for Upgrade Targets and Different Damping Coefficients.

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